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Three-Dimensional Light Intensity Distribution Model for an Elliptical Photoreactor

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Matsuura, Cassano, and Smith (*11*) and Matsuura and Smith (*12*) proposed a diffuse light model to represent the distribution of light intensity within an elliptical reflector-tubular photoreactor. The model was selected because it represented the author's intuitive concept of the light distribution resulting from an imperfect reflector better than did the classical radial distribution model (*2, 3, 4, 5, 6, 7, 14*). Data presented by Jacob and Dranoff (*8*) support the view that a diffuse model might be more appropriate than a radial model in some cases.

It is the purpose of this note to propose a three-dimensional diffuse model to represent the light distribution in an elliptical photoreactor. The model contains two adjustable parameters whose values can be estimated from experimental data. It is shown that the diffuse light model of Smith and coworkers represents a limiting case of the three-dimensional model when one of its adjustable parameters assumes an extreme value.

THE MODEL

This model, shown schematically in Figure 1, consists of two concentric right cylinders; the inner cylinder of radius R_r represents the reactor tube while the outer cylinder of radius R_1 represents the light source. The light source cylinder is assumed to be a continuous sheath of light of uniform intensity.

Following the methods for point kernel integration, similar to those used in nuclear radiation shielding problems (*9*), the light intensity at any point in the reactor is given by

$$I(r, z') = \int_0^{z'} \int_0^{2\pi} \frac{S_0 R_1 e^{-\mu x}\, d\psi dz}{R_1^2 + r^2 + z^2 - 2R_1 r\cos\psi} + \int_0^{L-z'} \int_0^{2\pi} \frac{S_0 R_1 e^{-\mu x}\, d\psi dz}{R_1^2 + r^2 + z^2 - 2R_1 r\cos\psi} \quad (1)$$

The model can be used to compute the radial and axial variation of intensity at any point within the reactor. It was assumed that no reflection or refraction occurs at the wall, that the light is monochromatic, and that the medium

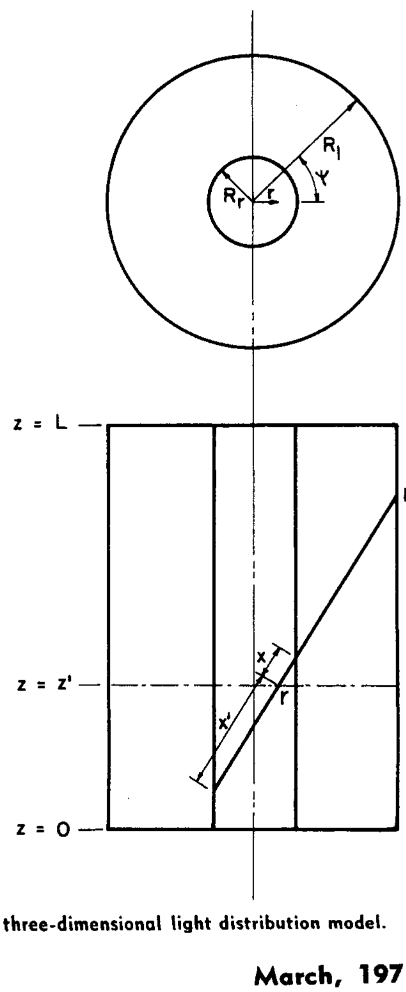

Fig. 1. Schematic of three-dimensional light distribution model.

between the light source and reactor does not absorb at the wavelength considered. Polychromatic light sources can be incorporated into the model with slight modification. For illustrative purposes it is proposed here with monochromatic light only.

The model contains two adjustable parameters, R_1 and S_0. The value of R_1 used in the model governs the axial distribution of light intensity. Ideally, R_1 is selected so as to match the model closely to experimental axial light distribution data. If such data are not available, R_1 can be made arbitrarily large so as to approximate a flat axial profile.

The light intensity I_w directed into the reactor from a solid angle of 180 deg. can be determined experimentally by means of an optically dense actinometer such as the potassium ferrioxalate system (*1, 10, 13*). The use of such actinometers insures that essentially all incident light is absorbed in a region near the wall. The source intensity S_0 can be adjusted so that the incident intensity, integrated over the reactor length, agrees with the experimental intensity determined by actinometry.

A LIMITING MODEL

The model given by Smith and coworkers (*11, 12*) is a limiting case of the three-dimensional model. This case occurs when the source radius R_1 is allowed to become infinitely large.

As $R_1 \to \infty$, the line $\overline{r\,r'} \to R_1$, and consequently the intensity is given by

$$\lim_{R_1\to\infty} I(r, z') = \int_0^{z'} \int_0^{2\pi} \frac{S_0 e^{-\mu x} d\psi dz}{R_1} + \int_0^{L-z'} \int_0^{2\pi} \frac{S_0 e^{-\mu x} d\psi dz}{R_1} \quad (2)$$

As $R_1 \to \infty$, x is measured normal to the axial centerline of the reactor and therefore becomes independent of z. The flux is then given by

$$\lim_{R_1\to\infty} I(r) = \frac{L\,S_0}{R_1} \int_0^{2\pi} e^{-\mu x}\, d\psi \quad (3)$$

For convenience the coordinate system is shifted by moving the origin from the axial centerline to r where the angular direction is now measured by θ. The length x is given in terms of the new coordinates by

$$x = -r\cos\theta + (R_r^2 - r^2 \sin^2\theta)^{1/2} \quad (4)$$

Instead of integrating on θ from 0 to 2π, the integration in Equation (3) can be carried out in two steps to yield

$$\lim_{R_1\to\infty} I(r) = \frac{L\,S_0}{R_1} \int_{-\pi/2}^{\pi/2} (e^{-\mu x} + e^{-\mu x'})\, d\theta \quad (5)$$

where x', as shown in Figure 1, is an extension of $\overline{r\,r'}$ through the reactor and is given by

$$x' = r\cos\theta + (R_r^2 - r^2\sin^2\theta)^{1/2} \quad (6)$$

As the radius of the source R_1 becomes infinite, the maximum intensity within the reactor at $\mu = 0$ is equal to twice the incident intensity I_w. The ratio of adjustable parameters S_0/R_1 appearing in Equation (3) can be determined by forcing the flux within the reactor to equal $2I_w$ as required at $\mu = 0$.

$$\lim_{\substack{\mu\to 0\\ R_1\to\infty}} I(r) = \frac{L\,S_0}{R_1} \int_{-\pi/2}^{\pi/2} 2\, d\theta = 2I_w \quad (7)$$

From Equation (7)

$$\frac{S_0}{R_1} = \frac{I_w}{\pi L} \quad (8)$$

Using Equations (5) and (8), the flux is given by

$$\lim_{R_1\to\infty} I(r) = \frac{I_w}{\pi} \int_{-\pi/2}^{\pi/2} (e^{-\mu x} + e^{-\mu x'})\, d\theta \quad (9)$$

The intensity distribution given in Equation (9) is identical to the diffuse light distribution model proposed by Matsuura and Smith (*12*) as Equation (9) in their work.

The above result establishes that the model proposed by Matsuura and Smith is a limiting case of the more general, three-dimensional model. In most cases it is expected that the limiting model would adequately represent light distribution in an elliptical photoreactor. In some cases where the geometric arrangement of the reactor-reflector system leads to strong axial variation of intensity, the three-dimensional model would better represent the actual light intensity distribution.

NOTATION

- I = light intensity, einstein/sq.cm.-sec.
- I_w = incident intensity directed into the reactor from a solid angle of 180 deg., einstein/sq.cm.-sec.
- L = length of distributed source, cm.
- r = radial distance, cm.
- $\overline{r\,r'}$ = line from point r inside reactor to point r' on surface of light source, $(R_1^2 + r^2 + z^2 - 2R_1 r\cos\psi)^{1/2}$
- R_1 = radius of source cylinder, cm.
- R_r = radius of reactor, cm.
- S_0 = source intensity, einsteins/sq.cm.-sec.
- x = distance through reactor measured along $\overline{r\,r'}$, cm.
- z = axial distance, cm.
- z' = axial position within reactor, cm.

Greek Letters

- α = molal absorptivity, sq.cm./g.-mole
- μ = attenuation coefficient, cm.$^{-1}$
- ψ = angle of rotation with coordinate center at axial centerline of reactor, radians
- θ = angle of rotation with coordinate center at radial position r, radians

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